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VECTORS AND ROTORS.

Vectors and Rotors, with Applications. By O. Henrici, Ph.D., LL.D., F.R.S., and G. C. Turner, B.Sc. Pp. xv + 204. (London : Edward Arnold, n.d.) Price 4s. 6d.

PROF. HENRICI can always be depended upon to embellish any mathematical subject which he touches, because, with the skill of the analyst, he combines the keen perception of the geometer, which ever seeks to render the results of analysis in some way visible by spatial representation—or, perhaps, to reach the results directly (and often more simply) without any aid from analysis at all. To a mathematician of this kind the subject of vector analysis is peculiarly appropriate. We are therefore indebted to Mr. Turner for putting into systematic form the lectures delivered by Prof. Henrici at the City and Guilds Technical College, and producing a very simple and elementary work the methods and ideas of which should find a very early introduction into our ordinary mathematical teaching.

The system here put forth is non-Hamiltonian. A vector is throughout a mere " carrier." With Hamilton it was this and more; every unit vector, when employed as a factor, said Hamilton, is to be regarded as a quadrantal versor the plane of which is perpendicular to the vector. In the non-Hamiltonian system the vector is not in any way associated with the notion of rotation. Some vectors are, except as regards *direction* and *sense,* absolutely unrestricted in space; others (such as forces acting on a body) are restricted to definite right lines and are called *localised vectors.* For these latter the special name of " rotors " has been invented, and Prof. Henrici must excuse an adherent of the Hamiltonian system for saying that this name seems to be wholly unjustified in a system which refuses to associate the notion of a rotational operation with any vector. Assuming that a " rotor " means, perchance, a " rotator," how comes it that such a name is applied to a mere " carrier "? There is another term also adopted by Prof. Henrici the justification of which is at least difficult, viz. the term " ort." A vector of unit length is called an " ort," which is explained to be " short for orientation," and " orientation " makes a dangerous suggestion of rotation. The " ort " is, of course, Hamilton's *unit vector.* The " rotor " and the " ort " should be regarded by anti-Hamiltonians as the trail of the serpent.[1]

The contrast between the two systems is well illustrated by the discussion of the product, $\alpha\beta$, of two vectors, α and β, which forms the subject of chapter iii. of Prof. Henrici's book. With Hamilton the nature of the expression follows simply and naturally; $\alpha\beta$ means α/β^{-1}, an operation implying rotation—the conversion of the vector β^{-1} into the vector α. It can therefore be taken as either a combined tensor and versor operation, or a combined scalar and vector operation. This at once gives us the complete specifi-

[1] Prof. A. Lodge suggests the term "locor" for rotor.

cation of the vector of $\alpha\beta$, and also that of the scalar of $\alpha\beta$, making the latter equal to $-ab \cos \theta$, where a and b are the tensors of α and β, and θ the angle between them.

Prof. Henrici, by a very simple and consistent rule, specifies the vector part and makes it identical with Hamilton's specification, but he makes the scalar $+ab \cos \theta$, by what, after all, amounts to a perfectly arbitrary and dogmatic definition (p. 95), its systematic connection with the mode of defining $V\alpha\beta$ being somewhat strained and unconvincing.

This, however, is a matter of no consequence, since he is quite at liberty to lay down his own definitions, inasmuch as he is not hampered by the Hamiltonian notion of rotation as associated with a vector.

As regards notation in this part of the subject, it may be pointed out that Prof. Henrici uses $[\alpha\beta]$ for the Hamiltonian $V\alpha\beta$, and (α,β) instead of $S\alpha\beta$, which certainly does not seem to be an improvement, especially when we have to write down a long vector or scalar equation—such, for example, as (iii.), p. 199. Again, the notation $[\alpha \,|\, \beta+\gamma]$, instead of $V\alpha(\beta+\gamma)$, is scarcely pleasing to the eye, even if it is not calculated to lead to slips in working.

The only indication that Prof. Henrici gives of his view of the quaternion system is found in p. 104, where he dispenses with the operation of division by vectors. " This operation is complicated and will not be considered at all. It leads to the much more complicated Theory of Quaternions." It is, however, quite open to a Hamiltonian to say nothing about division of vectors; he can treat his vectors as mere " carriers," and claim all the results of a non-Hamiltonian theory as his own ; for a non-Hamiltonian is not necessarily an anti-Hamiltonian theory. It remains, of course, quite true that with Hamilton division is the primary notion, and multiplication the secondary.

The subjects selected by Prof. Henrici for vector treatment are geometrical and statical. Almost all the prominent results of elementary geometry are shortly and neatly obtained, and among the illustrations of this subject are the Peaucellier and Hart mechanisms for the description of a right line. There is a very full discussion of centres of mass, and a planimetric method of finding the centre of mass of any area, which method is not so well known as it ought to be. The determination of the centre of parallel forces by the use of link (or funicular) polygons is fully explained, while—to the great advantage of the student—Prof. Henrici is very lavish of his figures.

So very few elegances escape the watchful eye of Prof. Henrici that one feels a pleasure in pointing out something that he might have included in his discussion of force systems. The centre of a parallel system of forces is known to everyone, but the *astatic centre* of a system of coplanar forces has received little attention. Yet it is a striking entity, and one which is closely allied to the other centre. Its definition is fairly well known; perhaps the best specification of it treats it as the point of intersection of the line of no moment with the line of no virial.

The portion of the book dealing with statics treats largely of the stresses in frameworks, shearing forces, bending moments, &c., the treatment being, of course,

all vectorial, that is, geometrical, and marked by great clearness of exposition. Such a treatment of statics forms a most needful corrective of the methods of a purely " analytical statics," which has a strong tendency to keep the subject aloof from reality, and to obscure its physical nature. " One does not find figures in this book," boasted Lagrange in his " Mécanique Analytique," but the absence of geometrical methods and conceptions is not to the advantage of the subject.

In the penultimate chapter Prof. Henrici gives a short, very useful, and well explained account of the reciprocal figures of graphic statics, and the last chapter is a very short one on the deduction of the elementary trigonometrical formulæ from vector methods. With all deference to the author, however, it is to be feared that pupils will not, within time at the earth's disposal, be so much accustomed to think in vectors as to deduce their notions of a sine and a cosine otherwise than by the old method.

Next to the systematic teaching of the solution of all kinds of equations by graphic constructions, the wider employment of geometrical methods in dynamics is our greatest desideratum, and for this reason we have to thank Prof. Henrici for this elegant little treatise.   GEORGE M. MINCHIN.

THREE PROTOZOAN ARTICLES.

A Treatise on Zoology. Edited by E. Ray Lankester, LL.D., F.R.S., &c. Part i. Introduction and Protozoa. Second Fascicle. Pp. vi + 451. (London : A. and C. Black, 1903.) Price 15s. net.

THE erratic order in which the various volumes of Prof. Lankester's treatise are appearing is, from the nature of their subject, a matter of very little consequence, and we are glad to welcome now this instalment of the protozoan chapter. It is the second fascicle of part i., of which the first fascicle, containing the introduction and the groups not here included, has still to appear. The inconvenience of the intended arrangement of parts is clearly demonstrated, and it is very fortunate that it has not resulted in the detention at the press of the valuable essays which make up this volume. A large part of the editor's difficulties have resulted, it is clear, from his adherence to the plan of producing bound volumes of nearly uniform size—in following, that is to say, the mode of publication of the recent " Cambridge Natural History " and of other similar works of collaboration. We believe it would prove to be in the interest of authors and readers alike if no attempt were made by the editors of series of this kind to produce periodically completed volumes, and if the separate articles were issued uniformly, but unbound, in the style of German monographs. The total expense to the purchaser of the whole series could remain the same by an obvious arrangement, while the gain to many specialists would be immense. We have a case in point in the present volume. Prof. Minchin's valuable monograph on the Sporozoa occupies about one-half of the whole volume, and might, we gather, have been already for some time in our hands if it had appeared separately in paper covers. Its subject is precisely one in which publication might well have been both early and individual

in the interests of the medical profession, for which it has, perhaps, its chief importance at the present time. The deliberate manufacture of volumes, as such, while we can see nothing at all to recommend it, is exposed at the same time to the serious objection of stimulating over-production. The publication of a complete " Cambridge Natural History," and now of what is virtually an Oxford treatise, suggests inevitably that among the whole body of English zoologists a good deal of research has been recently sacrificed to text-book writing, of which a large part, however conscientious, has been redundant.

We can say this now with the greater assurance, because it cannot be taken as applying to the excellent articles on the Foraminifera, the Sporozoa, the Ciliata, and the Acinetaria in the present volume. The section dealing with the Sporozoa, by Prof. Minchin, takes its place as an admirable systematic account of the group, prefaced by a general sketch of their characters and of the typical life-history. The recent developments of our knowledge of sporozoan parasites in connection with malarial disease give a special importance, as we have said, to this monograph. Prof. Minchin provides in his description of the Hæmosporidia exactly what is now becoming essential knowledge for the student of disease, and it is highly desirable, we think, that medical men should approach the study of this group from a more general point of view than that permitted in the restricted accounts of the malaria parasite written specially for their use. In the interests of further developments of curative and preventive treatment in new directions, it is of the first importance that the morphology and life-cycles of the members of this group should be completely determined, although, as the author claims, " the life-cycle of the malarial parasite is now thoroughly known in all its features." The recent work of Schaudinn, who has explained the occurrence of relapse in malaria without fresh infection as due to a kind of parthenogenetic reproduction by resistant and long-lived macrogametocytes, is an example of the value in these inquiries of a zoological outlook, and it is to be remembered that the " black spores " of Ross have not yet been assigned with certainty to their place in a life-cycle. With regard to the voluminously alleged connection between the Sporozoa and cancer, Prof. Minchin is content to express the hostility of most zoologists, but he gives all the necessary material for following the discussion elsewhere. In summing up the affinities of the whole group he decides against the theory of Euglenoid ancestry which Bütschli advanced, and argues in favour of a descent from the Rhizopoda, quoting the interesting example of parasitism which Schewiakoff has found in simple amœboid forms. He concludes his article with a valuable compilation of sporozoan hosts, including Labbé's list with modern additions, and an abundant bibliography is appended, brought up to the beginning of the present year. It would be difficult to suggest any improvement in the author's selection of illustrations or in their execution.

Prof. Hickson, who has undertaken the Infusoria, does not include the Flagellata, but deals only with the Ciliata and Acinetaria, grouped as the Corticata Heterokaryota. Here again we can have nothing but praise for his admirably illustrated account of these